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SHORTER COMMUNICATIONS

HEAT TRANSFER AND WALL FRICTION IN CONSTANT FLUX LAMINAR NATURAL CONVECTION OF WATER

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NOMENCLATURE

Nu, Nusselt number;
Gr^*, modified Grashof number;
Pr, Prandtl number;
C_f, friction coefficient;
μ, dynamic viscosity;
T, temperature;
h, heat transfer coefficient;
x, flow direction coordinate;
k, thermal conductivity;
q, heat flux;
ν, kinematic viscosity;
g, gravitational acceleration;
β, volume expansion coefficient;
L, total heater length;
τ_w, shear stress at wall.

Subscripts
x, local property or function;
f, film property;
ref, reference property;
∞, bulk property.

INTRODUCTION

THE CALCULATION of heat transfer rates by laminar natural convection from a uniform heat flux surface was obtained analytically by Sparrow and Gregg [1] for the case of a vertical flat surface. Hartnett and Welsh [2] experimentally verified the local heat transfer results of Sparrow and Gregg for convection from the inside of a vertical tube; Goldstein and Eckert [3] did the same for a flat plate, and recently Vliet and Lui [4] obtained similar results for a large flat plate where all investigators employed water as the test fluid. Sparrow and Gregg [5] have shown analytically that the heat transfer results for constant wall temperature flat plates and cylinders are in close agreement for all but the smallest diameter cylinders for gases with Prandtl numbers of 0·72 and 1·0. Thus it would be expected that with a properly sized cylinder the heat transfer results for a constant heat flux circular cylinder in water should agree with the constant heat flux flat plate solution of Sparrow and Gregg [1].

Part of this study was motivated by the goal of experimentally examining the heat flux–shear stress relationship in turbulent natural convection. Hence, there was a need to develop an experimental technique for measuring wall shear stress in natural convection. Since an exact theory was available for the laminar case [1], it was felt that the validity of any such method should first be verified by its agreement with the laminar theory. As a result this study was directed towards simultaneous measurements of the local film coefficient and the wall shear stresses in laminar natural convection from a vertical surface.

EXPERIMENTAL APPARATUS AND METHOD

The experimental study required apparatus and instrumentation to model the uniform flux wall, measure and record local values of the heat flux and wall temperature, provide the bulk liquid, and to measure the wall shear stress.

Heater

The heater was constructed with five separately controlled heating sections which could be adjusted to provide a uniform wall heat flux resulting in a wall temperature variation in the flow direction. The heater elements required some development and finally were made by winding nichrome wire on ceramic cylinders. A machineable ceramic was threaded and the nichrome wire was machine-wound onto the ceramic. The ceramic and wire were fired to about 1650°F after each of several thin applications of sodium silicate solution. This solution insulated the individual windings from one another. The heater elements were then placed inside a stainless steel tube to form the composite heater. A thin mica sheet was placed between the heaters and tube wall to insure good thermal contact and concentricity. The heaters were supported from the bottom by an L-shaped support. The heater is shown in Fig. 1.

Several tests were performed with the heater which gave confidence that it modeled the uniform heat flux case. First the heater was tested in a horizontal position rotating it through five positions so that each of the five thermocouples was in the up position. These tests were performed in air and water. The air test showed only a $\frac{1}{4}$ degree F variation for a ΔT_w of 100 degree F while the scatter was $\frac{1}{2}$ degree F in water at $q = 500$ Btu/hft^2. Some data were taken with the heater upside down, or with the leads taken out the top. Results from these tests were within the experimental scatter for all tests taken in the normal position. Data were taken on the effect of the trailing edge on heat transfer by using a guard heater. No heat transfer effects were noticed with and without the guard heater. Because of the difficulty in modeling uniform heat flux near the leading edge, the $x = x_1$ position was located approximately $\frac{1}{2}$ in. from the leading edge.

Heat flux and wall temperature record

The measurement of wall temperature was achieved by using copper–constantan thermocouples silver-soldered to the bottom of drilled wells in the heater wall at various distances along the flow direction. The thermocouples and thermocouple circuit were calibrated as placed in the system. The recorded temperature data were taken using a VIDAR digital integrating recording voltmeter. Individual

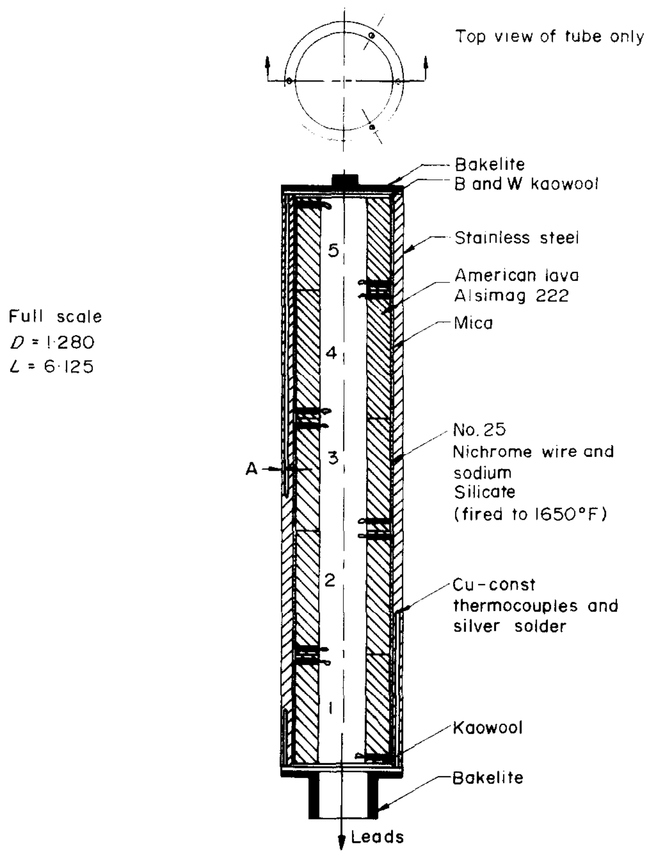

FIG. 1. Sketch of heater.

wattmeters were used to measure the heat flux values. The line voltage was kept constant by voltage regulation.

Thermal stratification was minimized for the 6 in. heater. A thermocouple rake was used to continuously monitor the temperature variation of the bulk fluid along the axis of the cylinder. Thermal stratification was kept below 1°F for the 6 in. heater. This effect was accounted for in the experimental value $(T_w - T_\infty)_x$ where $(T_w - T_\infty)_x$ was defined as

$$(T_w - T_\infty)_x = T_{w,x} - T_{\infty,x}$$

so that the temperature difference between the wall and heater surface was calculated with the local values of the temperature.

Measurement of wall shear stress

The measurement of the wall shear stress was accomplished using a device patterned somewhat after that used by Cavendish in 1798 to evaluate the Universal Gravitational Constant. The device used either the twist of a wire or a direct counter balance weight to measure the average wall shear force. The heater was pivoted on knife edges at one end of a balance arm in the center of a long (approximately 40 ft) piece of music wire. Using standard boundary layer assumptions, a force balance on the heater showed that the total wall shear force could be measured by:

1. establishing an initial balance position in the heat transfer medium,
2. turning on power and balancing any deflections detected by the measuring arm; or by
3. counteracting the force of the total wall shear by the twist of a long wire until the original position was re-established.

Under steady conditions the amount of twist of the wire measured the total wall shear force.

The leads were taken from the heater along a stationary wire opposite the twisted wire. The deflection of the measuring arm was detected by a simple scheme. A columnated light beam was directed onto a mirror located on the opposite end of the measuring arm from the heater. The reflected beam was projected onto a screen and the zero-point was located by a fine line drawn on the screen. The accuracy of the device was established as $\pm 0{\cdot}02$ g in $0{\cdot}2$ g by measuring unknowns. A 0·02 g weight represented the smallest weight which gave a noticeable deflection of the measuring arm.

EXPERIMENTAL RESULTS

Heat transfer result

The heat transfer data are shown below in Fig. 2. All fluid properties were evaluated at the local film temperature $T_{f,x} = (T_w + T_\infty)_x/2$. In Fig. 2 the Nusselt number is defined as

$$Nu_x = \frac{h_x\, x}{k_f}$$

or

$$Nu_x = \frac{q_{\text{wall}}\, x}{\Delta T_{w,x}\, k}$$

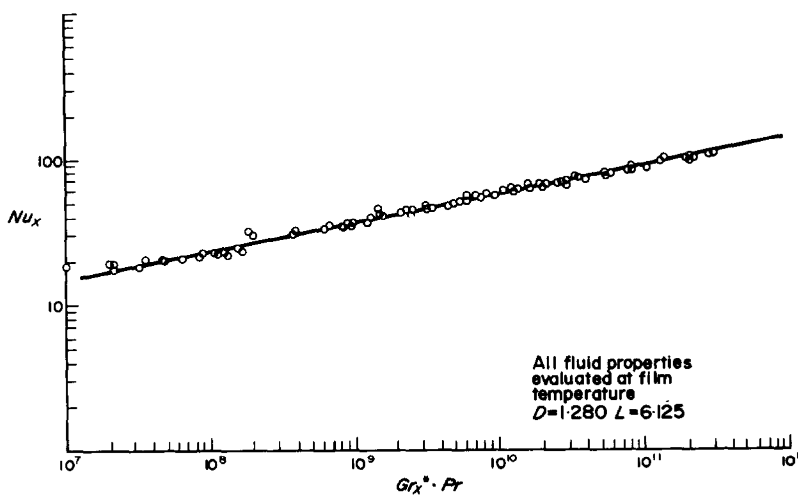

FIG. 2. Laminar natural convection with a uniform wall heat flux.

and the modified Rayleigh number is

$$Ra_x^* = \frac{g\beta q_{w,x}{}^4}{\nu^2 k}.\,Pr.$$

The solid line is based on the flat plate results of Sparrow and Gregg. These results predict a wall temperature variation

$$\frac{(T_{w,x} - T_\infty)}{(T_{w,L} - T_\infty)} = \frac{x^{1/5}}{L} \tag{1}$$

along the plate. The experimental data of this study verified their analysis. A linear regression correlation of the data gave

$$Nu_x = 0{\cdot}600\, Gr_x^{*0.190}\, Pr^{0.316} \tag{2}$$

with an average error of 4·26 per cent for all of the 120 data points. An error analysis [6] gave $\Delta Nu_x/Nu_x = 5$ per cent.

From the previous figures one can see that the uniform heat flux flat plate analysis of Sparrow and Gregg adequately describes the heat transfer from the cylinder used in this study.

Wall friction results

In Fig. 3 below the measured values of the laminar friction coefficient defined as

$$\bar{C}_f = \frac{(1/L)\int_0^L \tau w_x\, dx}{(\mu_w \nu_f/L^2)} \tag{3}$$

are plotted against the modified Grashof number evaluated at $x = L$ for laminar flow. Mean values of the friction coefficient are used since the experimental apparatus could only measure the total shear stress. The experimental error is shown for each point. The analytical result calculated from the Sparrow and Gregg work is shown as the shaded area. The shaded area represents the difference in using the wall and bulk fluid temperature to evaluate the fluid properties.

The effect of the trailing edge on total shear measurements was investigated briefly. A 1·50 in. cylindrical guard heater was placed downstream, but not touching, the test heater. Shadowgraph observation of the boundary layer showed that no unusual flow patterns were caused by the guard heater. Since no measurable differences were noted with and without the guard heater, the effect is within the error band shown in Fig. 3.

CONCLUSIONS

Excellent agreement with the analytical work of Sparrow and Gregg has been shown for a wide range of modified Grashof numbers for both heat transfer and wall friction. The film temperature has been used as a reference temperature for water as the bulk fluid with success. A direct measurement of the wall shear stress has been demonstrated for the laminar regime which gives confidence to a future extension of the technique to the transition and fully turbulent regions.

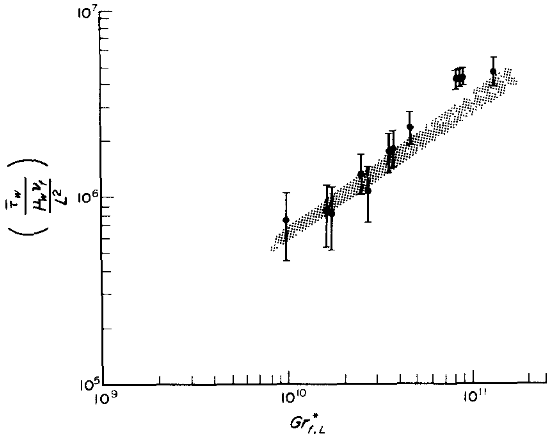

FIG. 3. Graph of experimentally determined skin-friction coefficients for laminar flow.

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